OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE **4721/01**

MATHEMATICS

Core Mathematics 1

THURSDAY 15 MAY 2008 Morning

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- **You are not permitted to use a calculator in this paper.**

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- **You are reminded of the need for clear presentation in your answers.**

WARNING

You are not allowed to use a calculator in this paper.

This document consists of **4** printed pages.

1 Express each of the following in the form 4^n:

(i) $\frac{1}{16}$, [1]

(ii) 64, [1]

(iii) 8. [2]

2 **(i)** The curve $y = x^2$ is translated 2 units in the positive x-direction. Find the equation of the curve after it has been translated. [2]

(ii) The curve $y = x^3 - 4$ is reflected in the x-axis. Find the equation of the curve after it has been reflected. [1]

3 Express each of the following in the form $k\sqrt{2}$, where k is an integer:

(i) $\sqrt{200}$, [1]

(ii) $\dfrac{12}{\sqrt{2}}$, [1]

(iii) $5\sqrt{8} - 3\sqrt{2}$. [2]

4 Solve the equation $2x - 7x^{\frac{1}{2}} + 3 = 0$. [5]

5 Find the gradient of the curve $y = 8\sqrt{x} + x$ at the point whose x-coordinate is 9. [5]

6 **(i)** Expand and simplify $(x - 5)(x + 2)(x + 5)$. [3]

(ii) Sketch the curve $y = (x - 5)(x + 2)(x + 5)$, giving the coordinates of the points where the curve crosses the axes. [3]

7 Solve the inequalities

(i) $8 < 3x - 2 < 11$, [3]

(ii) $y^2 + 2y \geqslant 0$. [4]

8 The curve $y = x^3 - kx^2 + x - 3$ has two stationary points.

(i) Find $\dfrac{dy}{dx}$. [2]

(ii) Given that there is a stationary point when $x = 1$, find the value of k. [3]

(iii) Determine whether this stationary point is a minimum or maximum point. [2]

(iv) Find the x-coordinate of the other stationary point. [3]

9 **(i)** Find the equation of the circle with radius 10 and centre (2, 1), giving your answer in the form $x^2 + y^2 + ax + by + c = 0$. [3]

(ii) The circle passes through the point $(5, k)$ where $k > 0$. Find the value of k in the form $p + \sqrt{q}$. [3]

(iii) Determine, showing all working, whether the point $(-3, 9)$ lies inside or outside the circle. [3]

(iv) Find an equation of the tangent to the circle at the point (8, 9). [5]

10 **(i)** Express $2x^2 - 6x + 11$ in the form $p(x + q)^2 + r$. [4]

(ii) State the coordinates of the vertex of the curve $y = 2x^2 - 6x + 11$. [2]

(iii) Calculate the discriminant of $2x^2 - 6x + 11$. [2]

(iv) State the number of real roots of the equation $2x^2 - 6x + 11 = 0$. [1]

(v) Find the coordinates of the points of intersection of the curve $y = 2x^2 - 6x + 11$ and the line $7x + y = 14$. [5]

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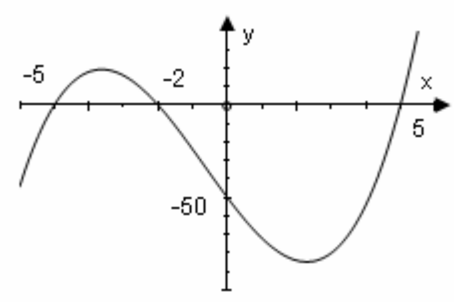
1	(i)	$n = -2$	B1 [1]	
	(ii)	$n = 3$	B1 [1]	
	(iii)		M1	$\sqrt{4^3}$ or $64^{\frac{1}{2}}$ or $\left(4^{\frac{1}{2}}\right)^3$ or $\left(4^3\right)^{\frac{1}{2}}$ or $4\times\sqrt{4}$ with brackets correct if used
		$n = \frac{3}{2}$	A1 [2]	
2	(i)		M1	$y = (x \pm 2)^2$
		$y = (x-2)^2$	A1 [2]	
	(ii)	$y = -(x^3 - 4)$	B1 [1]	oe
3	(i)	$\sqrt{2\times 100} = 10\sqrt{2}$	B1 [1]	
	(ii)	$\frac{12}{\sqrt{2}} = \frac{12\sqrt{2}}{2} = 6\sqrt{2}$	B1 [1]	
	(iii)		M1	Attempt to express $5\sqrt{8}$ in terms of $\sqrt{2}$
		$10\sqrt{2} - 3\sqrt{2} = 7\sqrt{2}$	A1 [2]	
4		$y = x^{\frac{1}{2}}$		
		$2y^2 - 7y + 3 = 0$	M1*	Use a substitution to obtain a quadratic or factorise into 2 brackets each containing $x^{\frac{1}{2}}$
		$(2y-1)(y-3) = 0$	M1dep	Correct method to solve a quadratic
		$y = \frac{1}{2}, y = 3$	A1	
			M1	Attempt to square to obtain x
		$x = \frac{1}{4}, x = 9$	A1	
			SR If first M1 not gained and 3 and ½ given as final answers, award B1 [5]	

5		**M1**	Attempt to differentiate
		A1	$kx^{-\frac{1}{2}}$
	$\frac{dy}{dx} = 4x^{-\frac{1}{2}} + 1$	**A1**	
	$= 4\left(\frac{1}{\sqrt{9}}\right) + 1$	**M1**	Correct substitution of $x = 9$ into their
$\frac{dy}{dx}$	$= \frac{7}{3}$	**A1**	$\frac{7}{3}$ only
		5	
6 (i)	$(x-5)(x+2)(x+5)$	**B1**	$x^2 - 3x - 10$ or $x^2 + 7x + 10$ or $x^2 - 25$ seen
	$= (x^2 - 3x - 10)(x+5)$	**M1**	Attempt to multiply a quadratic by a linear factor
	$= x^3 + 2x^2 - 25x - 50$	**A1**	
		3	
(ii)		**B1**	+ve cubic with 3 roots (not 3 line segments)
		B1√	(0, -50) labelled or indicated on y-axis
		B1	(–5, 0), (–2, 0), (5, 0) labelled or indicated on x-axis and no other x- intercepts
		3	
7 (i)	$8 < 3x - 2 < 11$	**M1**	2 equations or inequalities both dealing with all 3 terms resulting in $a < kx < b$
	$10 < 3x < 13$	**A1**	10 and 13 seen
	$\frac{10}{3} < x < \frac{13}{3}$	**A1**	
		3	
(ii)	$x(x+2) \geq 0$	**M1**	Correct method to solve a quadratic
		A1	0, –2
		M1	Correct method to solve inequality
	$x \geq 0, x \leq -2$	**A1**	
		4	

8 (i)	$\frac{dy}{dx} = 3x^2 - 2kx + 1$	**B1**	One term correct
		B1	Fully correct
		2	
(ii)	$3x^2 - 2kx + 1 = 0$ when $x = 1$	**M1**	their $\frac{dy}{dx} = 0$ soi
	$3 - 2k + 1 = 0$	**M1**	$x = 1$ substituted into their $\frac{dy}{dx} = 0$
	$k = 2$	**A1√**	
		3	
(iii)	$\frac{d^2y}{dx^2} = 6x - 4$	**M1**	Substitutes $x = 1$ into their $\frac{d^2y}{dx^2}$ and looks at sign
	When $x = 1$, $\frac{d^2y}{dx^2} > 0 \therefore$ min pt	**A1**	States minimum **CWO**
		2	
(iv)	$3x^2 - 4x + 1 = 0$	**M1**	their $\frac{dy}{dx} = 0$
	$(3x-1)(x-1) = 0$	**M1**	correct method to solve 3-term quadratic
	$x = \frac{1}{3}, x = 1$		
	$x = \frac{1}{3}$	**A1**	**WWW** at any stage
		3	

9	(i)		B1	$(x-2)^2$ and $(y-1)^2$ seen
		$(x-2)^2+(y-1)^2=100$	B1	$(x\pm 2)^2+(y\pm 1)^2=100$
		$x^2+y^2-4x-2y-95=0$	B1	correct form
			3	
	(ii)	$(5-2)^2+(k-1)^2=100$	M1	$x = 5$ substituted into their equation
		$(k-1)^2=91$ or $k^2-2k-90=0$	A1	correct, simplified quadratic in k (or y) obtained
		$k=1+\sqrt{91}$	A1	cao
			3	
	(iii)	distance from (–3, 9) to (2, 1)		
		$=\sqrt{(2--3)^2+(1-9)^2}$	M1	Uses $(x_2-x_1)^2+(y_2-y_1)^2$
		$=\sqrt{25+64}$	A1	
		$=\sqrt{89}$		
		$\sqrt{89}<10$ so point is inside	B1	compares their distance with 10 and makes consistent conclusion
			3	
	(iv)	gradient of radius $=\frac{9-1}{8-2}$	M1	uses $\frac{y_2-y_1}{x_2-x_1}$
		$=\frac{4}{3}$	A1	oe
		gradient of tangent $=-\frac{3}{4}$	B1√	oe
		$y-9=-\frac{3}{4}(x-8)$	M1	correct equation of straight line through (8, 9), any non-zero gradient
		$y-9=-\frac{3}{4}x+6$		
		$y=-\frac{3}{4}x+15$	A1	oe 3 term equation
			5	

10 (i)	$2(x^2-3x)+11$	**B1**	$p=2$
	$=2\left[\left(x-\frac{3}{2}\right)^2-\frac{9}{4}\right]+11$	**B1**	$q=-\frac{3}{2}$
	$=2\left(x-\frac{3}{2}\right)^2+\frac{13}{2}$	**M1**	$r=11-2q^2$ or $\frac{11}{2}-q^2$
		A1	$r=\frac{13}{2}$
		4	
(ii)	$\left(\frac{3}{2},\frac{13}{2}\right)$	**B1√**	
		B1√	
		2	
(iii)	$36-4\times2\times11$	**M1**	uses b^2-4ac
	$=-52$	**A1**	
		2	
(iv)	0 real roots	**B1**	cao
		1	
(v)	$2x^2-6x+11=14-7x$	**M1***	substitute for *x*/*y* or attempt to get an equation in 1 variable only
	$2x^2+x-3=0$	**A1**	obtain correct 3 term quadratic
	$(2x+3)(x-1)=0$	**M1dep**	correct method to solve 3 term quadratic
	$x=-\frac{3}{2},\ x=1$	**A1**	
	$y=\frac{49}{2},\ y=7$	**A1**	
			SR If A0 A0, one correct pair of values, spotted or from correct factorisation **www** **B1**
		5	